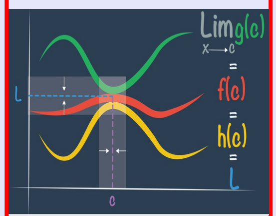
Math 261

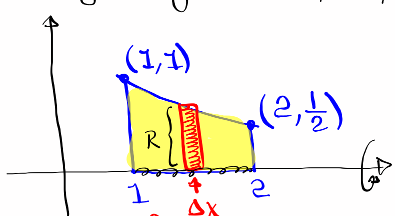
Fall 2022

Lecture 47



Feb 19-8:47 AM

Draw an enclosed region by $x=1$, $x=2$, $y=0$, and $y=\frac{1}{x}$.



Find its area.

$$A = \int_1^2 [\text{Top} - \text{Bottom}]\,dx$$

$$= \int_1^2 \left(\frac{1}{x} - 0\right)dx = \underbrace{\int_1^2 \frac{1}{x}\,dx}_{\text{Calc 2}}$$

$$= \ln x \Big|_1^2$$

$$= \ln 2 - \overset{0}{\cancel{\ln 1}} = \boxed{\ln 2}\ \checkmark$$

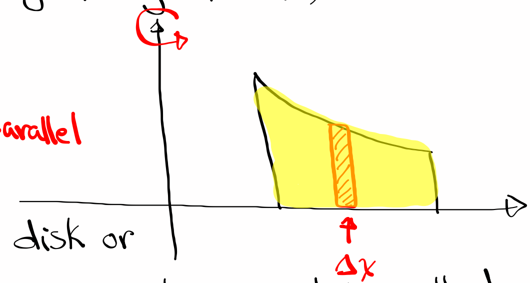
Rotate by x-axis, Find Volume. Disk

$$V = \int_1^2 \pi\left[\left(\frac{1}{x}\right)^2\right]dx = \pi\int_1^2 x^{-2}\,dx = \pi \cdot \frac{x^{-1}}{-1}\Big|_1^2 = -\pi \cdot \frac{1}{x}\Big|_1^2$$

$$= -\pi\left[\frac{1}{2} - 1\right] = \boxed{\frac{\pi}{2}}$$

Nov 21-9:30 AM

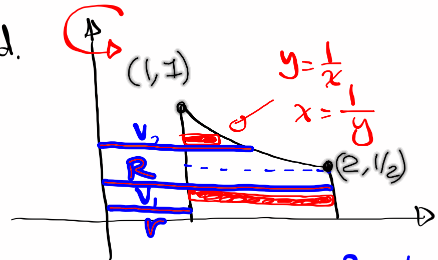
Rotate the region by y-axis, and find its Volume.



Ref. Rec. is parallel to A.O.R.

We cannot use disk or washer method. We must use shell method.

I don't know shell Method.



$$V_1 = \int_0^{1/2} \pi \left[2^2 - 1^2\right] dy$$

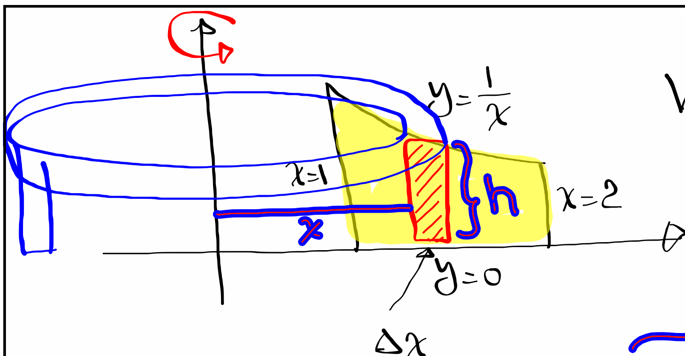
$$V_2 = \int_{1/2}^{1} \pi \left[\left(\frac{1}{y}\right)^2 - 1^2\right] dy$$

$V = 2\pi$

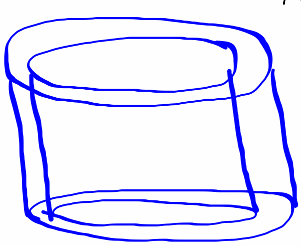
Make Sure to finish this.

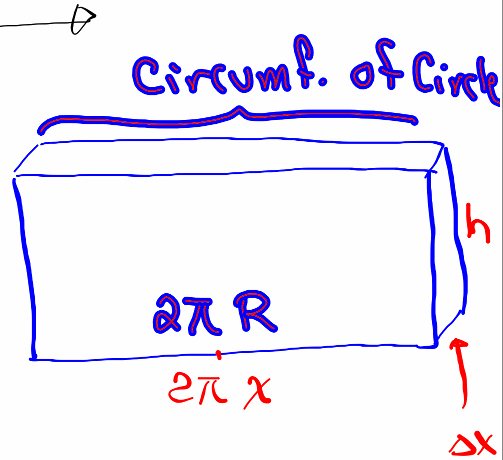
$V = V_1 + V_2$

Nov 21-9:39 AM



$$V = \int_1^2 2\pi x \cdot \frac{1}{x}\, dx$$





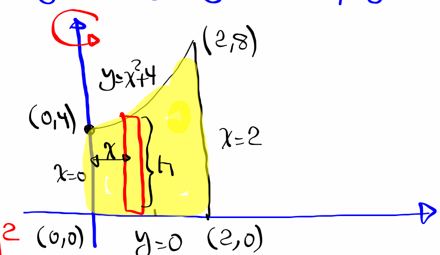
$$V = \int_1^2 2\pi x \cdot \frac{1}{x} dx = \int_1^2 2\pi \, dx$$

$$= 2\pi x \Big|_1^2 = 2\pi(2-1) = \boxed{2\pi}$$

$V = 2\pi x \cdot h \cdot \Delta x$

Nov 22-8:49 AM

Draw the enclosed region by $y = x^2+4$, $y=0$, $x=0$, $x=2$.



$$A = \int_0^2 [x^2+4-0]\,dx$$

$$= \int_0^2 (x^2+4)\,dx = \left(\frac{x^3}{3}+4x\right)\Big|_0^2 = \frac{8}{3}+8 = \boxed{\frac{32}{3}}$$

Rotate this region by the y-axis, find the Volume

Ref. Rec is parallel to A.O.R.

Shell Method $\quad V = \int_0^2 2\pi(\text{How far from A.O.R})(\text{Height of R.R.})\,dx$

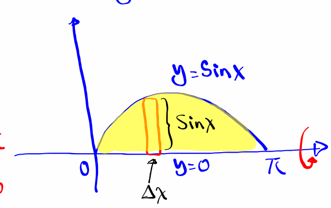
$$V = 2\pi\int_0^2 (\ x\ )(x^2+4)\,dx = 2\pi\int_0^2 (x^3+4x)\,dx$$

$$= 2\pi\left[\frac{x^4}{4}+2x^2\right]\Big|_0^2 = 2\pi(4+8) = \boxed{24\pi}$$

Nov 22-8:55 AM

Draw the enclosed region by $y=0$, $y=\sin x$ for $0 \le x \le \pi$.

$$A = \int_0^{\pi} (\sin x - 0)\,dx$$



$$= \int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi}$$

$$= -[\cos\pi - \cos 0] = -[-1-1] = \boxed{2}$$

Rotate about x-axis, find Volume.

$\underbrace{\text{Region 100\% attached to A.O.R., Ref. Rec. } \perp \text{ A.O.R.}}_{\text{Disk Method}}$

$$V = \int_0^{\pi} \pi[\sin x]^2\,dx = \pi\int_0^{\pi} \sin^2 x\,dx$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad = \pi\int_0^{\pi} \frac{1-\cos 2x}{2}\,dx$$

$$\cos 2x = \cos^2 x - \sin^2 x \qquad = \frac{\pi}{2}\left[x - \frac{1}{2}\sin 2x\right]\Big|_0^{\pi}$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

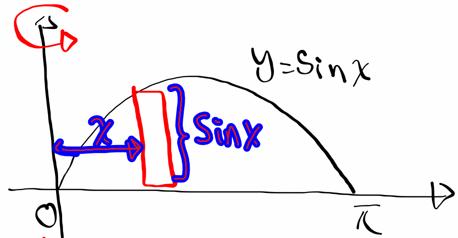
$$= \frac{\pi}{2}\left[\pi - \cancel{\tfrac{1}{2}\sin 2\pi}^{\,0} - \cancel{0}^{\,0} + \cancel{\tfrac{1}{2}\sin 0}^{\,0}\right]$$

$$= \boxed{\frac{\pi^2}{2}}$$

Nov 22-9:04 AM

Now rotate about Y-axis.

$\underbrace{\text{Ref. Rec. } \parallel \text{ A.O.R.}}_{\text{Shell}}$



$$V=\int_0^{\pi} 2\pi\left(\begin{array}{c}\text{How far}\\ \text{from}\\ \text{A.O.R}\end{array}\right)\left(\begin{array}{c}\text{height}\\ \text{of}\\ \text{Ref. Rec.}\end{array}\right)dx$$

$$=2\pi\int_0^{\pi} \underbrace{x \sin x}_{\text{Calc. 2}}\, dx = 2\pi\left[-x\cos x+\sin x\right]\Big|_0^{\pi}$$

Integration by Parts

$$=2\pi\left[-\pi\cos\pi+\sin\pi-0\right]$$

$$=2\pi\left[-\pi(-1)\right]=\boxed{2\pi^2}$$

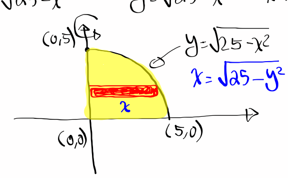
Nov 22-9:15 AM

Rotate the region in QI enclosed by $f(x)=\sqrt{25-x^2}$ by Y-axis, then find its Volume.

$f(x)=\sqrt{25-x^2}$ $\quad y=\sqrt{25-x^2}$ $\quad x^2+y^2=25$

Zhang's method

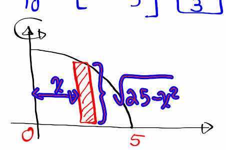
Disk



$$V=\int_0^5 \pi\left[\sqrt{25-y^2}\right]^2 dy$$

$$=\int_0^5 \pi(25-y^2)\,dy=\pi\left[25y-\frac{y^3}{3}\right]\Big|_0^5=\pi\left[125-\frac{125}{3}\right]=\boxed{\frac{250\pi}{3}}$$

Ref. Rec. || A.O.R.
Shell



$$V=\int_0^5 2\pi\cdot x\cdot\sqrt{25-x^2}\,dx$$

$u=25-x^2$
$du=-2x\,dx$
$-du=2x\,dx$

$x=0 \to u=25$
$x=0 \to u=0$

$$V=\pi\int_{25}^{0}\sqrt{u}\cdot -du=\pi\int_0^{25} u^{1/2}\,du$$

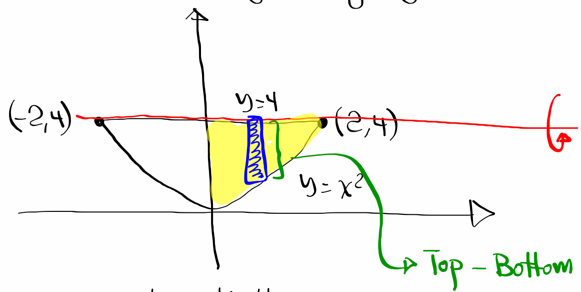
$$=\pi\frac{u^{3/2}}{3/2}\Big|_0^{25}=\frac{2\pi}{3}\left[25^{3/2}-0\right]$$

$$=\frac{2\pi}{3}\cdot 125$$

$$=\boxed{\frac{250\pi}{3}}$$

Nov 22-9:21 AM

Consider the enclosed region by $y=x^2$ and $y=4$.



$4-x^2$

Rotate this region by $y=4$.

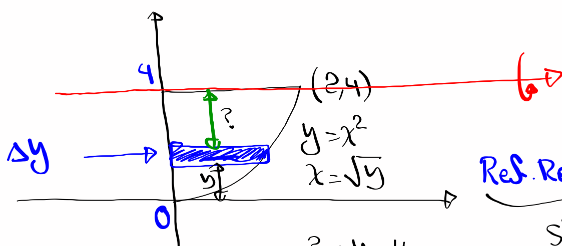
$$V = 2\int_0^2 \pi\left[4-x^2\right]^2 dx = 2\pi\int_0^2 (16-8x^2+x^4)\,dx$$

$$= 2\pi\left[16x - \frac{8x^3}{3} + \frac{x^5}{5}\right]\Bigg|_0^2$$

$$= 2\pi\left[32 - \frac{64}{3} + \frac{32}{5}\right]$$

$$= 2\pi\cdot\frac{256}{15} = \boxed{\frac{512\pi}{15}}$$

Nov 22-9:34 AM



Ref. Rec. || A.O.R.

$\underbrace{\hphantom{Ref. Rec. || A.O.R.}}_{\text{Shell}}$

$? + y = 4$

$? = 4 - y$

$$V = 2\int_0^4 2\pi\cdot(4-y)\cdot\sqrt{y}\,dy = 4\pi\int_0^4 (4y^{1/2} - y^{3/2})\,dy$$

$$= 4\pi\left[4\cdot\frac{y^{3/2}}{3/2} - \frac{y^{5/2}}{5/2}\right]\Bigg|_0^4 = 4\pi\left[\frac{8}{3}y\sqrt{y} - \frac{2}{5}y^2\sqrt{y}\right]\Bigg|_0^4$$

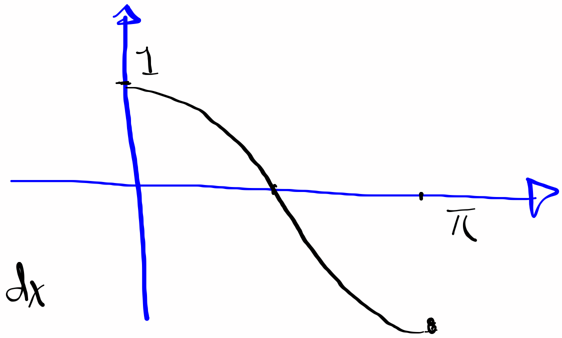
$$= 4\pi\left[\frac{8}{3}\cdot 4\cdot 2 - \frac{2}{5}\cdot 16\cdot 2\right] = 4\pi\left[\frac{64}{3} - \frac{64}{5}\right]$$

$$= 4\pi\cdot 64\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$= 256\pi\left[\frac{2}{15}\right] = \boxed{\frac{512\pi}{15}}$$

Nov 22-9:42 AM

Find f_{ave} for $f(x) = \cos x$ for $[0, \pi]$



$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} \cos x\,dx$$

$$= \frac{1}{\pi} \cdot \sin x \Big|_0^{\pi} = \frac{1}{\pi}\left[\sin\pi - \sin 0\right]$$

$$= \frac{1}{\pi}\left[0 - 0\right] = \boxed{0}$$

Nov 22-9:52 AM